COGNITIVE DEMANDS OF MATHEMATICAL INSTRUCTIONAL TASKS

Lower-Level Demands

Memorization

What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?

Expected Student Response:

$\frac{1}{2} = .5 = 50\%$

$\frac{1}{4} = .25 = 25\%$

Procedures without connections

Convert the fraction $\frac{3}{8}$ to a decimal and a percent.

Expected Student Response:

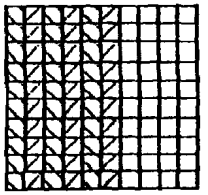
Fraction	Decimal	Percent
$\frac{3}{8}$	$8\overline{)3.000}$ .375, 24, 60, 56, 40, 40	.375 = 37.5%

Higher-Level Demands

Procedures With Connections

Using a 10 x 10 grid, identify the decimal and percent equivalents of $\frac{3}{5}$.

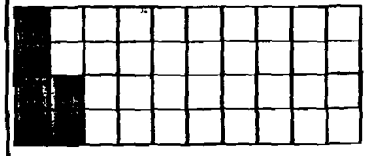
Expected Student Response:

Pictorial	Fraction	Decimal	Percent
	$\frac{60}{100} = \frac{3}{5}$	$\frac{60}{100} = .60$	.60 = 60%

Doing Mathematics

Shade 6 small squares in a 4 x 10 rectangle. Using the rectangle, explain how to determine each of the following: a) the percent of area that is shaded, b) the decimal part of area that is shaded, and c) the fractional part of area that is shaded.

One Possible Student Response:



a) One column will be 10% since there are 10 columns. So four squares is 10%. Then 2 squares is half a column and half of 10% which is 5%. So the 6 shaded blocks equal 10% plus 5% or 15%.

b) One column will be .10 since there are 10 columns. The second column has only 2 squares shaded so that would be one half of .10 which is .05. So the 6 shaded blocks equal .1 plus .05 which equals .15.

c) Six shaded squares out of 40 squares is $\frac{6}{40}$ which reduces to $\frac{3}{20}$.

Lower-level vs. higher-level approaches to the task of determining the relationships among different representations of fractional quantities (Stein & Smith, 1998).

THE TASK ANALYSIS GUIDE

Lower-Level Demands

Memorization Tasks

- involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
- cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- are not ambiguous—such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.

Procedures Without Connections Tasks

- are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- have no connection to the concepts or meaning that underlie the procedure being used.
- are focused on producing correct answers rather than developing mathematical understanding.
- require no explanations, or explanations that focus solely on describing the procedure that was used.

Higher-Level Demands

Procedures With Connections Tasks

- focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

Doing Mathematics Tasks

- require complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- demand self-monitoring or self-regulation of one's own cognitive processes.
- require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

The characteristics of mathematical tasks at each of the four levels of cognitive demand (Stein & Smith, 1998).